

QUADRATIC EQUATIONS

An equation in which the highest power of variable, i.e., degree of equation is 2, is called quadratic equation.

The general form of quadratic equations is -

$$ax^2 + bx + c = 0$$

where,

$$a \neq 0$$

a, b, c are real numbers

For e.g.,

$$x^2 + 3x + 2 = 0 \quad ; \quad x^2 - 4 = 0$$

$$(x+3)(x-1) = 0 \quad \text{etc.}$$

$$(x+1)^2 = 2(x-3)$$

$$\rightarrow x^2 + 2x + 1 = 2x - 6$$

$$x^2 + 2x - 2x = -6 - 1$$

$$x^2 = -7$$

$$x^2 + 7 = 0$$

∴ It is a quadratic equation.

$$(i) x^2 - 2x = (-2)(3-x)$$

$$\rightarrow x^2 - 2x = -6 + 2x$$

$$x^2 - 2x - 2x = -6$$

$$x^2 - 4x + 7 = 0$$

∴ It is a quadratic equation.

$$(ii) (x-2)(x+1) = (x-1)(x+3)$$

$$\rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$$

$$x^2 - x - 2 = x^2 + 2x - 3$$

$$x^2 - x^2 - x - 2x - 2 + 3 = 0$$

$$-3x + 1 = 0$$

∴ It is not a quadratic equation.



2) Represent the following situations in the form of quadratic equations:

i) The area of rectangular plot is 528m^2 . The length of the plot is one more than twice its breadth. we need to find the



length and breadth of the plot.

Let the breadth of rectangular plot be x .

According to given condition

$$\text{breadth} = x \text{ m}$$

$$\text{length} = (2x + 1) \text{ m}$$

∴ Area of Rectangular plot = $l \times b$

$$528 = (2x + 1) \times x$$

$$528 = 2x^2 + x$$

$$\therefore 2x^2 + x - 528 = 0 \text{ is the}$$

required ~~and~~ solutions

The product of two consecutive positive integers is 306. We need to find ~~the~~ integers.

Let first integer be x .

∴ According ~~to~~ second consecutive integer

will be $x + 1$

∴ According to given condition



$$x \times (x + 1) = 306$$

$$x^2 + x = 306$$

∴ $x^2 + x - 306 = 0$ will be required solutions

Rohan's mother is 26 years older than him.
The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Let Rohan's present age be x yrs.

∴ His mother's age = $x + 26$ yrs

3 years from now,

Rohan's age = $x + 3$

Age of Rohan's mother will be = $x + 26 + 3$

= $x + 29$

∴ The product of their ages 3 years from now will be 360 so that,

$$(x + 3)(x + 29) = 360$$

$$x^2 + 29x + 3x + 87 = 360$$

$$x^2 + 32x - 273 = 0$$

SOLUTION OF QUADRATIC EQUATION BY FACTORISATION

The values of variables which satisfy the quadratic equation is called roots of quadratic equation or its solution.

NOTE :- If a product of two numbers is '0' then either first or second number is '0'



(ii)

$$2x^2 + x - 6 = 0$$

$$\rightarrow 2x^2 + 4x - 3x - 6 = 0$$

$$2x(x+2) - 3(x+2) = 0$$

$$(x+2)(2x-3) = 0$$

$$x+2 = 0 \quad \text{or} \quad 2x-3 = 0$$

$$\circ \circ \quad x = -2 \quad \text{or} \quad 2x = 3$$

$$x = 3/2$$

(iii)

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$x(\sqrt{2}x+5) + \sqrt{2}(\sqrt{2}x+5) = 0$$

$$(\sqrt{2}x+5)(x+\sqrt{2}) = 0$$

$$\sqrt{2}x+5 = 0 \quad \text{or} \quad x+\sqrt{2} = 0$$

$$x = -5/\sqrt{2} \quad \text{or} \quad x = -\sqrt{2}$$

Q1) Find two consecutive positive integers, sum of whose squares is 365.

Let two consecutive positive integers be, x , $(x+1)$

According to given condition,

$$x^2 + (x+1)^2 = 365$$

$$x^2 + x^2 + 2x + 1 - 365 = 0$$

$$2x^2 + 2x - 364 = 0$$

Divide by 2,



$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x+14) - 13(x+14) = 0$$

$$(x+14)(x-13) = 0$$

$$\therefore x+14 = 0 \quad \text{or} \quad x-13 = 0$$

$$x = -14 \quad \text{or} \quad x = 13$$

$$\therefore x \neq -14$$

$$\therefore x = 13$$

\therefore Two consecutive integers are
~~13 & 14~~



SOLUTION OF QUADRATIC EQUATION BY
COMPLETING THE SQUARE



$$2x^2 - 7x + 3 = 0$$

Given equation is,

$$2x^2 - 7x + 3 = 0$$

$$2(x^2 - 7x/2 + 3/2) = 0$$

$$x^2 - 7x/2 + 3/2 = 0 \quad [\text{To get third term}]$$

$$x^2 - 7x/2 = -3/2 \quad \left(\frac{1}{2}x - \frac{7}{2}\right)^2$$

$$x^2 - 7x/2 + 49/16 = -3/2 + 49/16$$

$$(x - 7/4)^2 = 25/16$$

$$x - 7/4 = \pm \sqrt{25/16} = \pm 5/4$$

$$\therefore x - 7/4 = 5/4 \quad \text{or} \quad x - 7/4 = -5/4$$

$$x = 5/4 + 7/4 \quad \text{or} \quad x = -5/4 + 7/4$$

$$x = 3 \quad \& \quad x = 1/2$$



Q.E formula,

$$ax^2 + bx + c = 0$$

$$x^2 + (b/a)x + c/a = 0$$

$$x^2 + (b/2a)x + (b/2a)x + c/a = 0$$

$$x^2 + 2(b/2a)x + (b/2a)^2 - (b/2a)^2 + c/a = 0$$

$$(x + b/2a)^2 - (b^2 - 4ac/4a^2) = 0$$

$$\text{If } b^2 - 4ac \geq 0,$$

$$\Rightarrow x + b/2a = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = b^2 - 4ac$$

Roots of Equation

$$\text{If } b^2 - 4ac > 0 \text{ (Positive)}$$

Real & Unequal Roots

$$\text{If } b^2 - 4ac = 0$$

Real & Equal Roots

$$b^2 - 4ac < 0 \text{ (Negative)}$$

Roots are not real

$$a(x-p)(x-q) \geq 0$$

$$a(x^2 - (p+q)x + pq) = 0$$

$$-a(p+q) = b \quad \& \quad apq = c$$

$$\Rightarrow \begin{matrix} p+q = \frac{-b}{a} & \rightarrow \text{Sum of roots} \\ \leftarrow (\alpha+\beta) \end{matrix}$$

$$\begin{matrix} pq = \frac{c}{a} & \rightarrow \text{product of roots} \\ \leftarrow (\alpha\beta) \end{matrix}$$

Find the roots of the following equations

$$x - 1/x = 3, x \neq 0$$

$$x - 1/x = 3$$

$$x^2 - 1 = 3x \quad (\text{multiply by } x)$$

compare equation by
 $ax^2 + bx + c = 0$

$$\therefore a = 1, b = -3, c = -1$$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2 \times 1}$$

$$x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$$\therefore x = \frac{3 + \sqrt{13}}{2} \quad \& \quad x = \frac{3 - \sqrt{13}}{2}$$

NATURE OF ROOTS

If $ax^2 + bx + c = 0$, i.e. a quadratic equation in x , then, the nature of roots can be determined using

" $b^2 - 4ac$ " is called as ^{delta} Discriminant (D), its denoted by " Δ "

$$\text{i.e. } \Delta = b^2 - 4ac$$

So, a quadratic equation $ax^2 + bx + c = 0$ has

- ① two distinct real roots, if $b^2 - 4ac > 0$
- ② two equal real roots, if $b^2 - 4ac = 0$,
- ③ no real roots, if $b^2 - 4ac < 0$

1] Find the nature of roots of the following quadratic equation. If real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

→ We know,

$$\Delta = b^2 - 4ac$$

$$= (-3)^2 - 4(2)(5)$$

$$= 9 - 40$$

$$= -31$$

∴ It has no real roots

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

→ $\Delta = b^2 - 4ac$

$$= (-4\sqrt{3})^2 - 4(3)(4)$$

$$= 48 - 48$$

$$= 0$$

∴ It has equal real roots

3] Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800m^2 . If so, find its length and breadth.

→ Let length be $2x$ and breadth be x

$$\therefore \text{Area} = 800\text{m}^2$$

$$(x)(2x) = 800$$

$$2x^2 = 800$$

$$x^2 = 400$$

$$x = \pm\sqrt{400}$$

$$x = 20$$

∴ Yes it is possible

$$\therefore \text{length} = 40\text{m}$$

$$\text{breadth} = 20\text{m}$$

